Linear Algebra

# Linear Systems

Linear equation- equation in the form

* Where , , … are all constants
* Where , , … are all variables

Linear system- set of one or more linear equations that have the same variable set (can have any number of equations an any number of variables)

Solution of a system- tuple of n values that satisfies each equation in the system

* Tuple- ordered list

Solution set- set of all solutions

* If set is empty, then the system has no solutions and is inconsistent
* If set has one or more solutions, then the system in constant
* If two systems have the same number of systems, they are the same

**Consistent System**

**inconsistent System**

A consistent system can have a finite number of solutions or an infinite number of solutions

# Matrix Notation

Coefficient matrix- a matrix formed by all coefficients of a system (all the a values)

* The coefficient matrix is size

Augmented matrix- coefficient matrix and an extra column for solution of each equation (all a and b values)

* The coefficient matrix is size

# Elementary Row Operations

1. Replace one row
2. Interchange- switch two rows around
3. Scaling- multiplying row by constant

ERO does not change the solution set

2 matrixes are equivalent if you can turn one into another through ERO

# Echelon Form

Conditions

1. Any zero row is at the bottom
2. The leading entry of each row is in a column to the right of the column in from of it
3. All entries in a column below a leading entry are zeros

Leading entry- first non-zero term in row

# Reduced Row Echelon Form

Conditions

1. Must be in echelon form
2. Leading entry of each non-zero row is 1
3. Each leading entry is the only nonzero element in the column

Pivot positions- the leading 1s when in reduced Row Echelon Form

Pivot columns- columns that contain leading 1s

Elements that correspond to pivot positions in RREF are the ones used to make other rows in their column zero

The row reduction algorithm can be used on ANY matrix not just augmented matrix of a system

In any system the number of pivots (not in the last column) correspond to the number of basic variables in the system

The variables that are not in a pivot column are free variables

~ means row equivalent to

If pivot is in right most column of augmented matrix, then system has no solutions (inconsistent)

* If there is a zero. Row and a row where picot is in the right most column the system has no solution

# Vector Equations

A vector can also be written as or as

Adding to matrixes- add each element to the corresponding element in the other matrix and then

**Adding Vectors**

Matrix multiplied by constant- multiply each item in the matrix by the constant. then

**Adding Vectors**

Any vector in the form is called the linear combination of and

So the linear combination of and is which looks like a system of equations

So using linear combination we can convert between systems and vectors

Solving a system is the same ass asking if linear combination of the coefficient matrix equal the right-hand side vector

Subset spanned- set of all possible linear combinations of vectors

* Written as
* - because it can form every vector on the XY plane
* - because it can form every vector on the line x=0

is in if there. Is a set of c values where

# The Matrix Equation

The vector equation is

* But it can also be expressed as

In general if are columns are A and are elements of the column vector then

This gives us a way to multiply matrix by a vector if the column of the first equals row of the other

* So if the two matrixes are sizes and then they can be multiplied together
* Row vector rule- multiply the first row of A by the first column of and this will result in a matrix of size

For a solution of to exist the vector must be a linear combination of the columns of A so must be in the

* Create the augmented matrix and get it in RRE. If there is a pivot point in each column then a solution always exists

For each in to have a solution each in is linear combination of columns of A